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The Impact of Rigid Spheres on Rubber

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Synopsis

The applicability of Hertz's theory of impact to the case of rigid spheres colliding with rubber has been studied over a wide range of impact conditions. In general, the theory is adequate; but in many practical cases, the influence of the finite thickness of the sample is significant. Using previously published data on the influence of thickness on static indentation measurements, this effect can be predicted and good agreement with experiment is found. Resilience measurements have been made over a range of impact conditions. The time of impact, rather than its severity as such, was found to be the factor governing the resilience for the unfilled materials studied, provided that there was no slip of the sample relative to its holder. Slippage can occur if the sample is too thin, due to the shear stresses developed at the rear surface, and this results in the observed resilience being anomalously low.

INTRODUCTION

The resilience of rubber is usually measured by the impact of a spherical striker on the rubber sample. The mechanics of impact are thus important in the interpretation of this measurement, particularly as there are effects which are only imperfectly understood, such as the influence of time of impact on resilience. Other complicating factors which can affect the measurement of resilience in practice are the variation in effective hardness of the sample due to its finite thickness, and the slippage of the back surface of the sample relative to its holder, giving rise to frictional energy losses. This paper describes work on the general problem of the impact of a sphere on rubber and on the basis of the results discusses the factors mentioned above.

HERTZ'S THEORY OF IMPACT

The general theory for the impact of two perfectly elastic spheres is due to Hertz and is given, for example, by Timoshenko and Goodier.[1] For a rigid sphere of mass m and radius r impacting with velocity v on a relatively soft semi-infinite flat pad, the time to reach maximum indentation T (half the time of impact) is given by

$$T^5 = 3.4125 \frac{m^2}{E^2 r v} \quad (1)$$

* Part of this work was carried out while A. G. Thomas was a Visiting Professor At Akron University, Ohio, U.S.A.

where E is Young's modulus of the rubber. The maximum indentation x_{max} is given by

$$x^5_{max} = 0.4944 \frac{m^2v^4}{E^2r} \tag{2}$$

and the maximum radius of the circle of contact is given by

$$a_{max} = (x_{max}r)^{1/2}.$$

The above theory is strictly valid only for small deformations. However, Waters[2] has carried out indentation measurements of a rigid sphere on a plane rubber sheet which suggests that there is little error on this account for indentations up to at least half the sphere radius.

EXPERIMENTAL

Experiments with a simple pendulum impacting on a rubber pad were used to test eq. (1). The general method is similar to that used by previous workers, who were, however, mainly concerned with glass or rigid plastics.[3,4] The experimental arrangement is shown in Figure 1. The pendulum had a length of about 65 cm, and the spheres were ball bearings ranging in size from $^1/_4$ in. to 2 in. in diameter (density 7.8 g/cc) together with a rigid plastic ball (diameter 3.5 cm and density 1.15 g/cc). The suspension was a 46 s.w.g. (0.061 mm diameter) copper wire for the lighter spheres and 40 s.w.g. (0.122 mm diameter) stainless steel for the heavier ones. The release mechanism was an electromagnet with a pointed pole piece so that no spin was imparted to the sphere on release. The rubber sample was a pad 6.5 cm in diameter and 2.5 cm thick stuck to a heavy metal block clamped to the bench. When hanging vertically, the sphere was just in contact with the rubber surface. The velocity of impact was calculated from the position from which the ball was released and was varied from about 3 to 170 cm/sec.

The surface of the rubber sample (and that of the plastic ball) was coated with colloidal graphite to produce a thin conducting layer, which

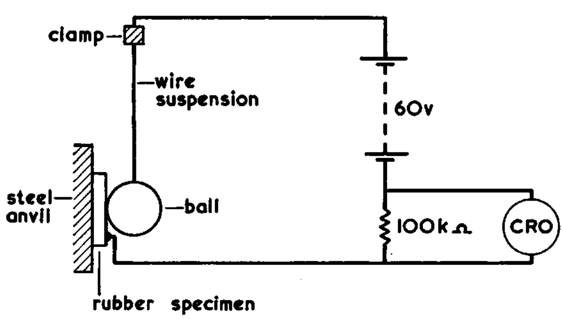

Fig. 1. Experimental arrangement using a simple pendulum to measure impact times and resilience. The surface of the rubber sample is coated with colloidal graphite to allow electrical conduction.

was found not to affect the resilience of the rubber. Using the circuit shown in Figure 1, a voltage was produced across the oscilloscope terminals when the ball was in contact with the rubber. This voltage triggered the time base, so that the time of impact could be deduced from the trace.

The resilience was calculated from the maximum amplitude of the rebounding ball and expressed as the percentage energy returned.

RESULTS AND DISCUSSION

Time of Impact

The results are shown in Figures 2 and 3 for three samples, the formulations of which are given in Table I. Also shown in Figures 2 and 3 are the theoretical lines derived from eq. (1) corresponding to the Young's modulus of each rubber. This was calculated from the rigidity modulus which was measured statically in a torsion apparatus using the actual samples employed in the impact measurements. The Young's modulus values were 9.5, 29.5, and 14 kg/cm^2 for the vulcanizates A, B, and C, respectively.

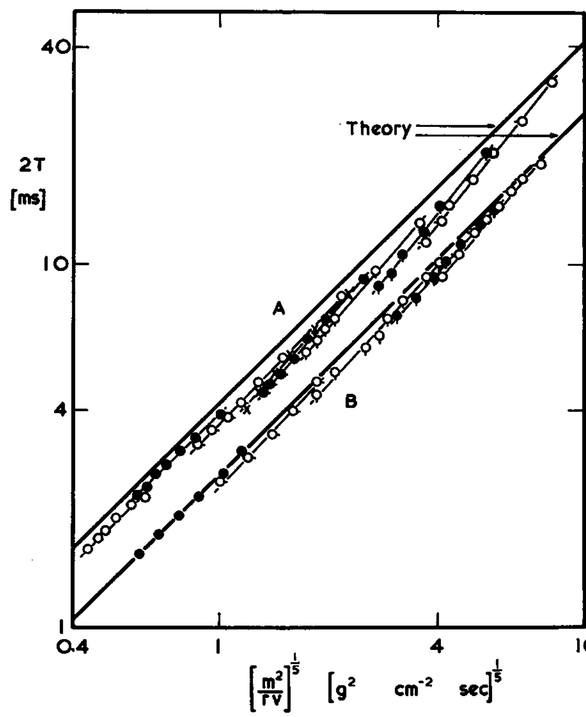

Fig. 2. Time of impact ($2T$) for the natural rubber vulcanizates A and B as a function of $(m^2/rv)^{1/5}$. The theoretical lines are calculated from eq. (1) using the experimentally determined Young's modulus values of 9.5 and 29.5 kg/cm^2, respectively. The symbols refer to the various balls used, diameters being given in inches. Steel balls: (○) $^1/_4$; (●) $^{11}/_{32}$; (○-) $^1/_2$; (●-) $^3/_4$; (ϙ) 1; (ϕ) 1$^1/_2$; (-○) 2. Plastic ball: (×).

TABLE I
Formulations of Samples Used[a]

Mix	A	B	C
Natural rubber	100	100	—
Butyl	—	—	100
Dicumyl peroxide	1	6	—
Zinc oxide	—	—	5
Sulfur	—	—	2
Stearic acid	—	—	3
Dibenzothiazyl disulfide	—	—	2
Tetramethylthiuram disulfide	—	—	1.5

[a] Vulcanization for 60 min at 150°C.

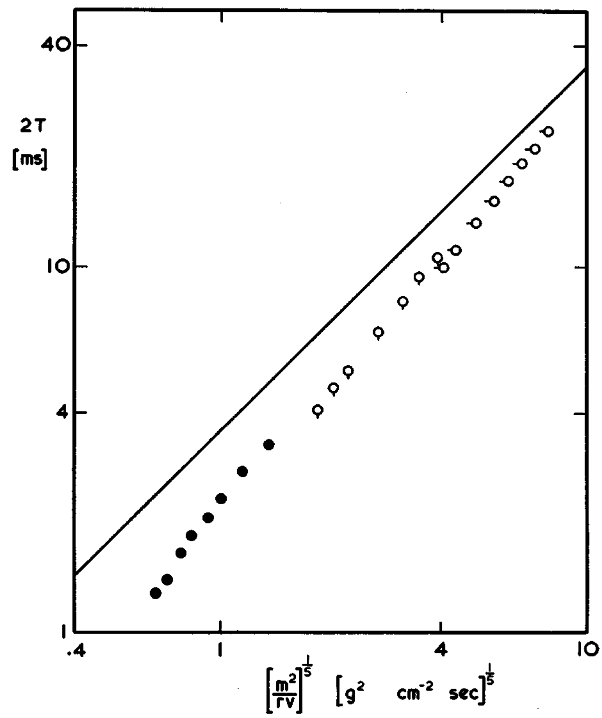

Fig. 3. Time of impact ($2T$) for the butyl vulcanizate C as a function of $(m^2/rv)^{1/5}$. The full line is calculated from eq. (1) using the statically determined Young's modulus value of 14 kg/cm². Symbols as in Fig. 2.

The results for the natural rubber vulcanizates A and B shown in Figure 2 are in broad agreement with the theoretical predictions both for the steel and plastic balls. Those for the butyl (Fig. 3) are not. The disagreement can be readily ascribed to the marked time dependence of the viscoelastic behavior of this material; the modulus under a relatively rapid test, as in an impact measurement, will be higher than for a longer term test, lasting several minutes, such as was used here to determine Young's modulus. The behavior of butyl will be discussed more fully when the resilience measurements are described; the time dependence of the modulus of the NR materials is much less pronounced, as they are much more resilient.

The results even for the natural rubber do, however, show systematic deviations from theory; the results for different-sized balls do not superpose well when plotted as in Figure 2. The reason for this is believed to be that the rubber blocks used, about 2.5 cm thick, were not thick enough to be considered semi-infinite as required by the theory. Waters[2] has shown that the static indentation of a rubber sheet by a sphere is influenced by the sheet thickness unless this is greater than about 5 times the radius of the circle of contact between the ball and the rubber. For the larger balls and the higher impact velocities, this condition was not met in the present experiments, especially for the softer rubber. Previous workers have studied the influence of thickness of the sample on impact behavior,[4] using hard plastics. In those cases, the effect of thickness is largely due to the flexural deformation of the sample. In the present experiments, where the rubber sample is stuck to a massive metal block, this effect will be negligible, as the indentation of the rubber is large compared with any flexure of the metal backing.

Influence of Sample Thickness on Time of Impact

On the basis of his results, Waters proposed an empirical equation which described the influence of thickness on static indentation. The thickness effect is governed by the parameter h/a, where a is the radius of the circle of contact and h is the thickness. The spatial extent of the strain distribution is determined by a, whereas the amount of indentation x, in the form of x/r, determines the severity of the strains.

Waters' relation can be expressed as

$$F = \frac{-16}{9} E r^{1/2} \left[\frac{x}{1 - \exp\,(-0.417h/a)} \right]^{3/2}$$

where F is the indenting force. The exponential term gives the effect of the finite thickness. The radius of the circle of contact a is related to the indentation x by the relation $a = (rx)^{1/2}$, and thus the differential equation governing the impact of a sphere on a rubber sheet can be written as

$$\frac{d^2x}{dt^2} = \frac{-16}{9} \frac{Er^{1/2}}{m} \left[\frac{x}{1 - \exp\,(-0.417hr^{-1/2}x^{-1/2})} \right]^{3/2}. \qquad (3)$$

To reduce this equation to a convenient nondimensional form, we define new variables $y = x/x_\infty$ and $\theta = t/T_\infty$, where x_∞ is the maximum indentation which would be attained on an infinite sheet, given by eq. (2), and T_∞ is similarly the time to maximum indentation on an infinite sheet, given by T in eq. (1). Equation (3) now becomes

$$\frac{d^2y}{d\theta^2} = -2.707 \left[\frac{y}{1 - \exp\,(-0.417ha_\infty^{-1}y^{-1/2})} \right]^{3/2} \qquad (4)$$

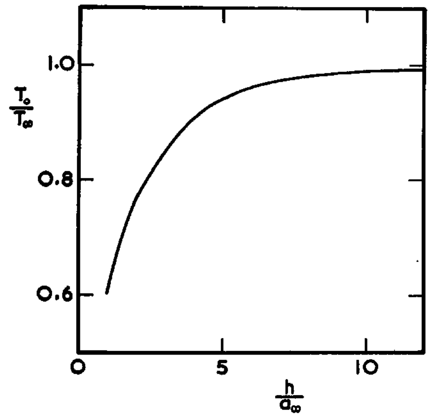

Fig. 4. Influence of thickness h on the time to maximum indentation T_0. T_∞ is the time to maximum indentation for an infinitely thick sheet and a_∞ is the maximum radius of the circle of contact for impact on an infinite sheet. Relation based on eq. (4).

where a_∞ is the maximum radius of the circle of contact for impact on an infinite sheet. The boundary conditions are that, when $y = 0$, $\theta = 0$ and $dy/d\theta = 1.4716$. The parameter h/a_∞ governs the thickness effect.

Numerical step-by-step integration of eq. (4) for various values of h/a_∞ yields the ratio of the time to maximum indentation T_0 to T_∞ in terms of h/a_∞. The results are shown in Figure 4. This indicates that, provided h/a_∞ is greater than 5, the time of impact is reduced by less than 5% from its value for an infinite block. However, this is not met by many of the results given in Figure 2.

Using the relation given in Figure 4, the values of the time of impact were corrected for the finite thickness of the samples, a_∞ being calculated from the known parameters (velocity, mass, and radius of the ball) for each impact measurement. Figure 5 shows the corrected values, T_∞, plotted as before against $(m^2/rv)^{1/5}$. The correction to the time of impact varies from a negligible amount for the small balls up to about 38% for the largest balls when impacting with the highest velocity on the soft rubber. The agreement with the theory is now substantially improved, the results for different balls superposing well. The remaining departures are probably little greater than the experimental error. A slight tendency for the experimental points to lie below the theoretical line would be expected at the shorter impact times, as the effective modulus of the rubbers, particularly the soft one, will be somewhat greater than the "static" modulus which was measured some minutes after loading and on which the theoretical line is based.

There is little value in correcting the results from the butyl vulcanizate for the thickness effect, as the departures from the simple theory, embodied in eq. (1), are clearly not due primarily to this cause. The departures are too great, of the wrong character, and, as noted above, probably associated with the low resilience and the related variation of modulus with frequency. A full treatment of the impact of a sphere on a viscoelastic material is beyond the scope of this paper.

A direct test of the effect of thickness was made using two standard resilience test devices, the Lupke pendulum and the Dunlop Tripsometer.[5] The former consists of a rod of mass 350 g with a spherical end of radius 0.63 cm, suspended so as to impact the rubber sample perpendicularly with a velocity of 140 cm/sec. The standard sample thickness for this device was 1.25 cm, but a double thickness sample (2.5 cm) was also used. The Dunlop Tripsometer consists essentially of an inertial disc turning on a low-

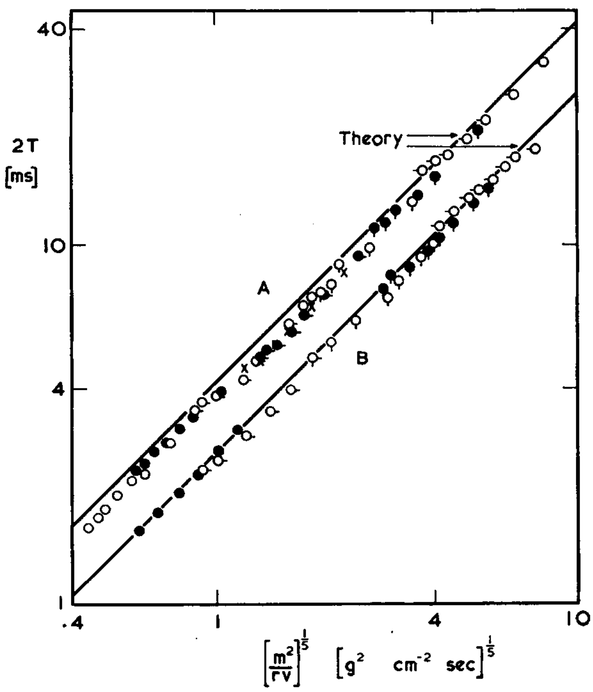

Fig. 5. Data from Fig. 2, but time of impact corrected for the influence of the finite thickness of the sample according to the theoretical curve in Fig. 4. Symbols as in Fig. 2. The lines are calculated from eq. (1).

friction bearing, with an indenter of radius 0.20 cm at its circumference. Subsidiary measurements on the machine actually used showed that the effective mass at the indentor was 5.22 kg, and the impacting velocity was 12.5 cm/sec. Two thicknesses of sample, 0.4 and 0.8 cm, were again used. The samples were made from the soft and hard vulcanizates A and B, respectively.

The theoretical impact times were calculated from the constants of the systems for infinitely thick samples and for the thicknesses employed. The impact times were measured electrically as previously described. Results are shown in Table II. The agreement between theory and experi-

TABLE II
Effect of Thickness on Time of Impact

Material	Thickness of sample, cm	Impact time, msec.	
		Theory	Experiment
	Lupke Pendulum		
A (soft)	∞	17.6	—
	2.5	15.4	15.7
	1.25	12.8	13.2
B (hard)	∞	11.2	—
	2.5	10.3	10.8
	1.25	8.7	8.9
	Dunlop Tripsometer		
A (soft)	∞	106	—
	0.8	88	90
	0.4	71	71
B (hard)	∞	67.2	—
	0.8	58.5	59
	0.4	48.4	50

ment is excellent, discrepancies being little greater than the experimental uncertainties.

The general conclusions from the above work on the impact time is that Hertz's solution is quite adequate for a sufficiently thick sample for the range of strains which occurred here. However, the sample thickness must be greater than 5 times the radius of the circle of contact if the effect of thickness is to be avoided. The theory based on Waters' results for static measurements adequately describes this effect.

Resilience

The resilience of the samples was calculated from the measured rebound of the ball from the rubber and expressed as the fractional energy returned.

Various combinations of impact velocity and ball sizes were used, and thus the impact time could be varied over a wide range; it was measured by the technique described above. Accurate resilience measurements were not possible over as wide a range of impact velocities as the impact time measurements. At the smallest velocities, the amplitude of the swings could not be measured sufficiently accurately; at the highest velocities, vibration of the supporting wire was troublesome.

Results for the hard NR vulcanizate B and the butyl vulcanizate C are shown in Figure 6 as a function of the time of impact. For C especially there is a marked dependence of resilience on impact time. However, the particular combinations of mass of ball and impact velocity appear to be unimportant. This implies that the materials are linear in their viscoelastic behavior for the strains developed. This is not surprising as the linear, small strain elastic theory was found to account well for the observed impact times.

The general dependence of resilience on impact time is consistent with the known variations of dynamic properties of rubbers with frequency of test in steady-state oscillatory measurements, the equivalent frequency being given[4] approximately by $(4T_0)^{-1}$.

The time of impact and the radius of the circle of contact a_∞ have been calculated for some resilience instruments in common use, and also for a

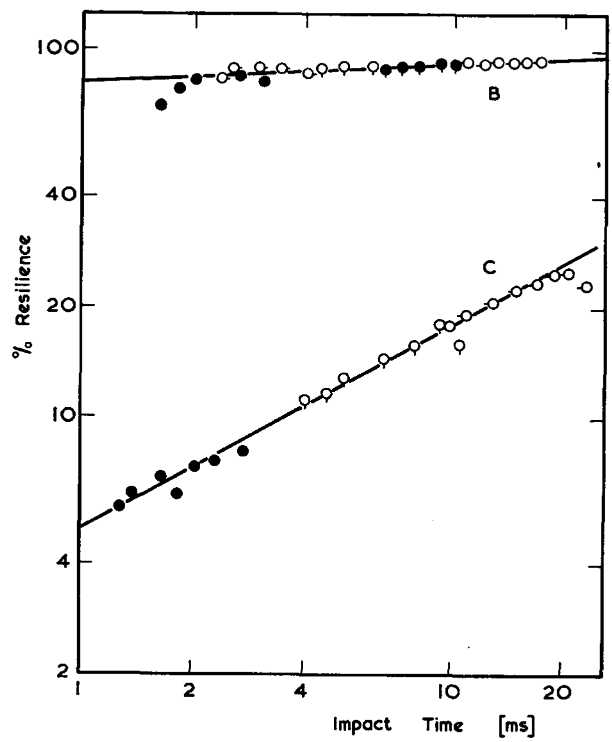

Fig. 6. Resilience as a function of time of impact for the natural rubber vulcanizate B and the butyl vulcanizate C. Symbols as in Fig. 2.

steel ball $^1/_{16}$ in. in diameter falling from a height of 10 cm, assuming a rubber with Young's modulus of 30 kg/cm^2 (equivalent to a British Standard or Shore hardness of about 55°). The equivalent masses were estimated from the apparatus dimensions. The results are shown in Table III. The times of impact vary widely and indicate that for some materials substantial differences will be found in their resilience measured on various instruments. The variation of resilience with time of impact is not the same for all materials, as indicated in Figure 6, and it is thus not possible easily to transform results from one instrument into those appropriate to another.

TABLE III
Calculated Values of Impact Time and Maximum Radius of the Circle of Contact a_∞

Instrument	Impact velocity, cm/sec	Impact time, msec	a_∞, cm	Recommended sample thickness,[5] cm
Dunlop Tripsometer	12.5	67	0.24	0.4–0.6
Dunlop Pendulum	122	23	1.1	2.5
Lupke Pendulum	140	11	0.58	1.25–1.65
Falling ball[a]	140	0.3	0.034	—

[a] $^1/_{16}$ in. diameter, 10-cm fall.

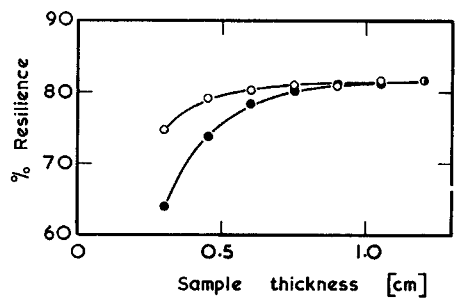

Fig. 7. Effect of sample thickness on apparent resilience of vulcanizate A measured by the Dunlop Tripsometer: (●) rear surface chalked; (O) rear surface clean.

The a_∞ values are such that, with the sample thicknesses usually employed, there will be a substantial effect of thickness on the behavior during impact, the h/a_∞ values being in the range 1.6 to 3. This indicates that substantial stresses are set up on the rear of the sample, and the shear stresses developed off the line of impact will tend to cause slippage of the sample in its holder if the friction is inadequate. This will lead to lower resilience values due to energy losses produced by the frictional movement. In principle, perfect lubrication as well as perfect adhesion would give zero losses from this cause, but the former case appears to be difficult to achieve in practice.

The influence of slippage has been shown in measurements on the Dunlop tripsometer using the soft rubber A in various thicknesses. In one set of measurements, the rear of the sample was chalked to reduce the friction artificially. The results, given in Figure 7, show that slippage at the rear is important for thicknesses below about 0.8 cm for this combination of instrument and rubber, and that the magnitude of the effect is strongly dependent on the friction between the sample and its holder. It is worth noting that, even in the absence of slip, the reduction in thickness will diminish the impact time, which itself leads to a lower resilience. The reduction observed with the clean-backed samples may well be due mainly to this cause, as the time of impact for the thinnest sample is about 0.6 of that for the infinitely thick case. It is obviously desirable in the design of resilience instruments that these effects are minimized by suitably choosing the impact conditions so that h/a_∞ is kept sufficiently large. A minimum value of 3 would appear to be adequate.

CONCLUSIONS

Hertz's theory of impact appears to be applicable to the impact of rigid spheres on rubber over a fair range of severities of impact. Thus the strains developed are still sufficiently small for small-strain linear elasticity theory to be sufficiently accurate. However, the finite thickness of the rubber sample gives rise, in many cases, to an important correction. A satisfactory theory has been developed, using Waters' results on static indentation, with which the appropriate correction can be calculated.

The dependence of resilience on impact conditions has been examined and found to be determined by the time of impact rather than severity as such, provided that the sample is sufficiently thick so that no slippage occurs relative to its holder. If this latter condition is not satisfied, spurious results may be obtained.

This work forms part of the research program of the Natural Rubber Producers' Research Association.

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